STANDARD &POOR'S STRUCTURED FINANCE

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Drill-Down Approach for Synthetic CDO Squared Transactions

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Standard & Poor's has developed a new approach to rating synthetic CDO transactions linked to a reference portfolio of CDOs and ABS — so-called "CDO squared" transactions.

These transactions have become very popular as market participants seek to compensate for tightening investment-grade corporate spreads.

A typical CDO squared transaction rated by Standard & Poor's might reference as many as 1,000 corporate names. However, given that the most liquid corporates in the credit default swap market number about 400, it is highly likely that each name will appear in more than one CDO tranche. Therefore, the default of one name could affect several CDO tranches. Fundamental to Standard & Poor's new approach is the ability to "drill down" to the corporates underlying each CDO tranche included within the CDO squared transaction. In this way, it is possible to accurately assess the effect on the CDO squared transaction of the overlap between CDO tranches.

This article describes in detail how the new approach addresses the key risks in CDO squared transactions, and presents a case study of the use of the approach on typical transactions. While this article is devoted to the analysis of synthetic transactions, the use of the drill-down approach for cash flow CDO squared transactions has also been developed, and will be the subject of a forthcoming article.

The CDO squared drill-down approach is the latest addition to the CDO Evaluator suite of tools, and is part of Version 2.3.

Key Benefits

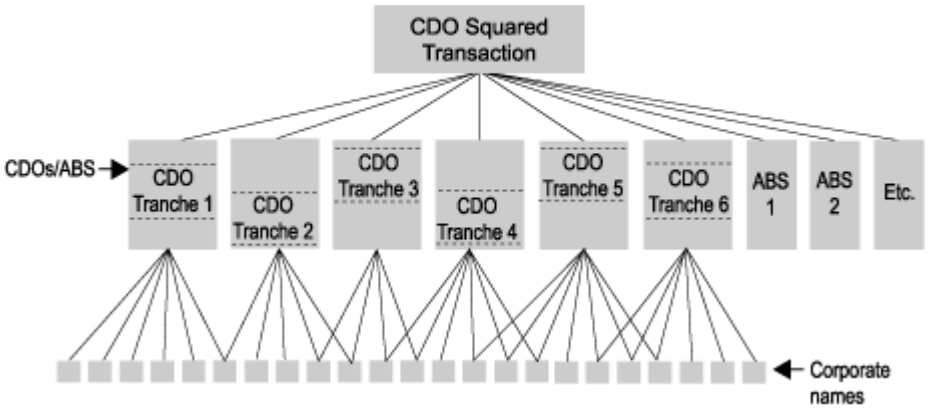
The key benefits of the drill-down approach are as follows:

- Correlation and overlap are modeled in an accurate and intuitive way, removing the need to estimate the correlation between pairs of CDO tranches.
- The introduction of asset-specific recoveries allows the recovery profile of each CDO tranche to be modeled dynamically, removing the need for CDO tranche recovery assumptions.
- The "attachment points" for the CDO squared transaction can be determined in a manner that is completely consistent with the terms and conditions of the transaction.

Structure of a Typical CDO Squared Transaction

CDO squared transactions are typically leveraged single-tranche CDOs, in which the underlying assets are CDO tranches, or a mixture of CDO tranches and ABS (see chart 1).

Chart 1
Structure of a Typical CDO Squared Transaction



Should a credit event occur on an underlying corporate name, a bidding process is typically used to establish a recovery value. The resulting loss is incurred by each CDO tranche referencing that corporate name. If total losses exceed the attachment point for the CDO tranche, a loss is incurred at the CDO squared level. The overall effect of a single corporate credit event will clearly depend on the number of CDO tranches referencing that corporate name — or the overlap among the underlying CDO tranches.

In most transactions, the occurrence of a loss to a CDO tranche from a corporate credit event does not trigger the default of the CDO tranche. Instead, losses incurred by the CDO tranche are simply passed through to the CDO squared transaction. Consequently, these transactions are typically structured to allow for the occurrence of multiple credit events within a given CDO tranche, or to allow "partial write-down". This is not necessarily the case for ABS tranches, where a default may immediately trigger a bidding process, with the resulting loss passed through to the CDO squared transaction.

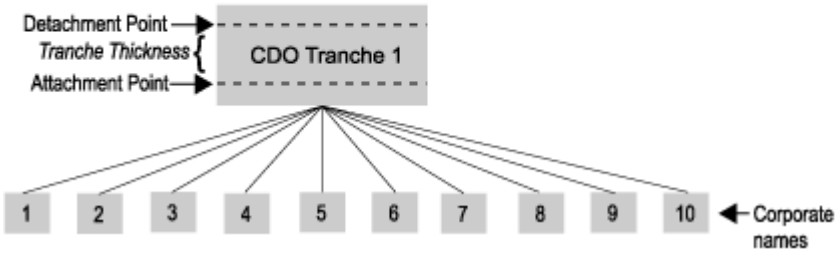
The Drill-Down Approach

Standard & Poor's has adopted the drill-down approach to capture the overlap and other risks in CDO squared transactions.

The technique involves the simulation of the correlated default and recovery of all assets in the CDO squared reference portfolio. In the case of CDO tranches, simulated net losses are passed through to each CDO tranche referencing a given underlying asset. When these net losses exceed the attachment point for that tranche, the excess net loss is passed on to the CDO squared transaction.

Net losses are capped by the "detachment point", also called the "exhaustion point". Investors in a CDO squared transaction will not be required to make payments for losses on an underlying CDO tranche that exceed the detachment point. Therefore, the maximum loss that can be passed on is equal to the difference between the attachment point and detachment point, or the tranche thickness (see chart 2).

Chart 2
Structure of a Single Underlying CDO Tranche



If assets other than drill-down CDO tranches (e.g., ABS) are included on a standalone basis in the CDO squared transaction, the correlated default of these assets is simulated in the same way as the corporate names underlying the CDO tranches. In the event of default of a standalone asset, a recovery assumption is applied to generate a net loss to the CDO squared transaction.

The result of the simulation exercise is a distribution of net losses experienced by the CDO squared transaction, from which rating-dependent attachment points can be obtained for the transaction.

CDO Attachment Points
The separate CDO tranches underlying a CDO squared transaction are generally not marketed individually — they are tailor-made for the CDO squared transaction. Nonetheless, an attachment point needs to be determined for each CDO tranche to be included in the CDO squared reference portfolio. These can be determined using Standard & Poor's CDO Evaluator (see Appendix).

For example, using the CDO Evaluator, a portfolio of 100 'A' rated uncorrelated corporate names with equally sized five-year exposures and recoveries equal to 35% would require a 'AA' attachment point equivalent to the default of four names. The final attachment point would be equal to 2.9%, calculated as four names * 1.11 adjustment factor * (100% – 35% recovery rate).

CDO Detachment Points
Once an attachment point has been sized, the arranger of the CDO squared transaction will determine the detachment point for each CDO tranche. This determines the thickness of each CDO tranche. In the example above, if each exposure equals €10 million, the 'AA' attachment point will be €29 million (2.9% * €1 billion). The detachment point will then be some number greater than €29 million. Assuming that the detachment point is set at €53 million, the thickness of the tranche will be equal to €24 million (€53 million – €29 million).

As a result, the number of credit events needed to reach the detachment point (once the attachment point is exceeded) is 3.7 credit events, calculated as €24 million / €10 million * (100% – 35%). The 35% recovery rate is included in the denominator because net losses are passed through to the CDO tranche. As illustrated below, while the attachment point determines the probability of default of the CDO tranche, the thickness of the tranche is crucial in determining the degree of loss that the tranche and ultimately the CDO squared transaction will suffer upon default. Consequently, while an arranger may set the detachment points in whatever manner he or she feels is appropriate, their position will have a significant effect on Standard & Poor's ratings on the CDO squared transaction.

Simulating Corporate/ABS Net Losses
The simulation of corporate and ABS net losses is the most straightforward part of Standard & Poor's new approach, as it is performed in an almost identical fashion to the use of the CDO Evaluator for plain-vanilla CDOs.

There is one important difference, however, namely the inclusion of asset-specific recoveries.

Asset-Specific Recoveries
When the CDO Evaluator is run for plain-vanilla CDOs, attachment points are determined by generating a distribution of correlated defaults for the portfolio, calculating scenario default rates (SDRs) from this distribution, and finally scaling the relevant SDR by the portfolio loss given default (LGD), equal to 100% minus the portfolio weighted-average recovery rate. In the new drill-down approach, however, a recovery assumption needs to be made for each asset in the portfolio. When an asset defaults, a net loss is recorded by multiplying the notional amount of exposure by 100% minus the recovery rate assumed for that asset. For example, if the notional amount of the exposure is €10 million, and the recovery assumption is 35%, the default will generate a net loss of €6.5 million, calculated as €10 million * (100% – 35%).

If all the recovery assumptions for the underlying assets are the same, then the use of asset-specific recoveries will give identical results to the scaling of the "gross" SDR by the portfolio LGD. If, however, there is some dispersion in the recoveries across the different assets, using asset-specific recoveries is likely to give different results. The reason is that recoveries are now being applied differently depending on which assets default, producing a "risk-adjusted" portfolio recovery rate. This may be lower than the weighted-average recovery rate, for example if lower-rated assets have lower recoveries than higher-rated assets.

Correlation, Default, and Recovery

The principal benefit of the drill-down approach is that each corporate name is explicitly linked to one or more CDO tranches. Therefore, when a corporate name experiences a credit event, and generates a net loss, this net loss is passed through to each CDO tranche referencing that name.

A standard approach used in portfolio credit risk analysis is to make some assumption about the correlation between pairs of assets. For corporate and ABS assets in the same country and sector, Standard & Poor's normally assumes a uniform asset correlation of 0.3 between the random numbers used to determine whether each asset defaults. In CDO squared transactions, however, there are three sources of correlation:

- Correlation between pairs of corporate or ABS assets within each underlying CDO tranche;
- Correlation between pairs of other standalone assets; and
- Correlation induced by the corporate name overlap between pairs of CDO tranches.

The last type of correlation is more difficult to quantify, as it depends both on the extent of the overlap and the level of industry concentration within each portfolio. However, because the drill-down approach models all three types of correlation in an accurate and intuitive way, there is no need to estimate the correlation between pairs of CDOs. A matrix is provided as an output of the CDO Evaluator (Version 2.3) to illustrate the extent of the overlap between pairs of CDOs.

Comparing Simulated Losses With Attachment/Detachment Points
As explained above, a net loss will be realized by each CDO tranche in each iteration of the simulation. If this net loss exceeds the attachment point of an underlying CDO tranche, that CDO tranche has in some sense "defaulted." In reality, however, the CDO tranche does not default — the excess loss is simply passed through to the CDO squared transaction.

For example, if the attachment point for a given CDO tranche is €29 million, and the net loss experienced by the CDO tranche is €35 million, then €6

million of losses will be passed through to the CDO squared transaction. Note that losses are capped by the detachment point. If, for example, the detachment point is set at €53 million, and the CDO tranche suffers net losses of €65 million, then only €24 million (€53 million – €29 million) of losses will be passed through to the CDO squared transaction.

Recoveries on the Underlying CDO Tranches

Given that the underlying CDO tranches cannot strictly default, the concept of a "recovery" is also somewhat misleading. Nevertheless, it is perfectly reasonable to consider the loss that a tranche will experience in the event that the attachment point is breached. This LGD is not a fixed number. It varies with every iteration of the model, depending on the level of simulated net loss experienced by the CDO tranche.

This is another advantage of the drill-down approach. There is no need to make a discrete recovery or LGD assumption for each underlying CDO tranche, as this is modeled dynamically through the introduction of asset-specific recoveries at the underlying corporate/ABS level.

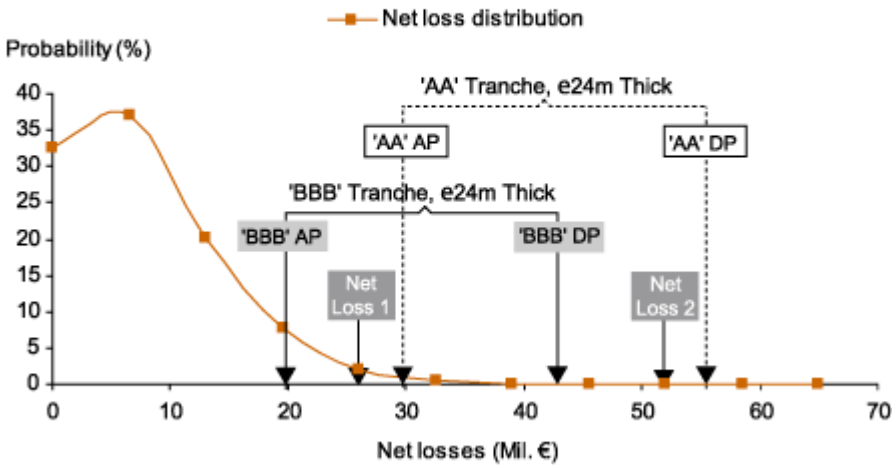
The Net Loss Distribution

Using the CDO Evaluator, the net loss distribution can be determined for the portfolio of corporate names with an average rating of 'A' and with a recovery rate of 35%. This distribution is represented by the curved line in chart 3. The 'BBB' and 'AA' attachment points are also shown (€19 million and €29 million, respectively), along with the relevant detachment points, assuming a tranche thickness of €24 million, as in the previous example. In addition, two examples of net losses (Net Loss 1 and Net Loss 2) from the portfolio are shown.

Chart 3

Examples of Simulated Net Loss 1 and 2 for Two Separate CDO Tranches

100 'A' corporate names, €10 million notional exposure per name, and 35% recovery



AP - Attachment point
DP - Detachment point

There are two important points to note about the net loss distribution:

- The area underneath the curve to the right of each attachment point expresses the probability that the attachment point is breached. This also represents the "default" probability of the CDO tranche. As expected, this area is much larger in the 'BBB' case. Therefore, the probability of default of the 'BBB' tranche is higher than that of the 'AA' tranche.
- In cases where the attachment point is breached, the shape of the distribution expresses the potential losses that could be passed through to the CDO squared transaction. In general, the fatter the tail of the net

loss distribution between the attachment point and detachment point of a tranche, the more severe the losses are likely to be. Thus, the likelihood of passing through large losses between the 'BBB' attachment and detachment points is greater than the corresponding likelihood for the 'AA' case.

The above points are in line with intuition, namely that both the probability of "default" and the LGD are much lower for higher rated tranches than for lower rated tranches.

Calculating LGD for a Tranche
Using the two net loss examples shown in chart 3, the percentage LGD can be easily calculated, as can the implicit "recovery" rates for each tranche.

At Net Loss 1:

(i) A net loss of €26 million, equivalent to four defaults, occurs.

(ii) The 'BBB' attachment point is breached, resulting in approximately €7 million in net losses being passed from the 'BBB' tranche through to the CDO squared transaction.

- LGD = €7 million / €24 million = 29%
- Recovery rate = 100% – 29% = 71%

(iii) The 'AA' attachment point is not breached, so no losses are passed from the 'AA' tranche through to the CDO squared transaction.

- LGD = 0%
- Recovery rate = 100%

At Net Loss 2:

(i) A net loss of €52 million, equivalent to eight defaults, occurs.

(ii) The 'BBB' attachment point is breached, and the 'BBB' detachment point is exceeded. Consequently, the entire 'BBB' tranche is lost. Net losses to be passed through to the CDO squared transaction from the 'BBB' tranche are capped by the tranche thickness of €24 million.

- LGD = €24 million / €24 million = 100%
- Recovery rate = 0%

(iii) The 'AA' attachment point is breached, and a €23 million net loss is passed from the 'AA' tranche through to the CDO squared transaction.

- LGD = €23 million / €24 million = 96%
- Recovery rate = 100% – 96% = 4%

Varying the Tranche Thickness
Using the 'AA' CDO tranche described above, the behavior of the LGD of the tranche as its thickness is varied can be investigated. If the attachment point of a tranche is kept constant, and the detachment point is increased, the LGD is expected to decrease and vice versa.

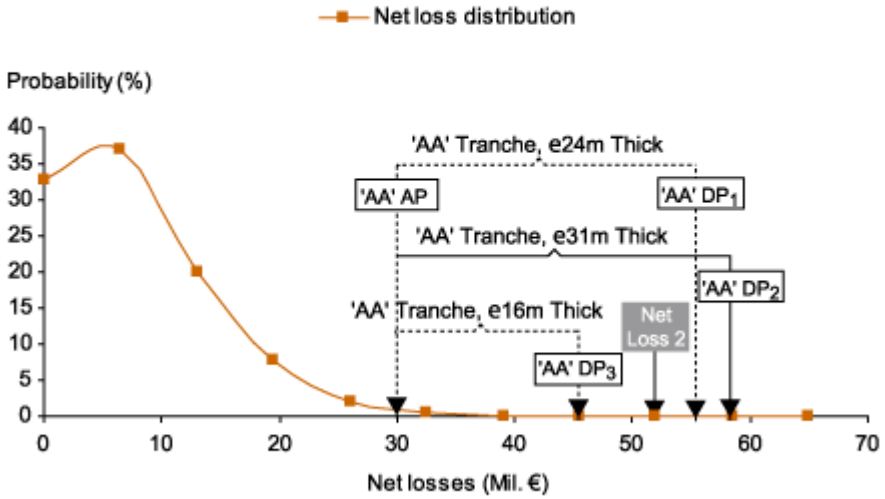
For example, in the case of Net Loss 2 above, if the 'AA' detachment point is increased to €60 million, the tranche thickness increases to €31 million (€60 million - €29 million), and the LGD decreases from 96% to 74% (€23 million / €31 million). If the detachment point is lowered to €45 million, the tranche

thickness decreases to €16 million (€45 million - €29 million) and the LGD increases from 96% to 100% (€23 million / €16 million, but capped at 100% by the tranche size). This is illustrated in chart 4.

Chart 4

Effect of Net Loss 2 When 'AA' Tranche Thickness is Varied

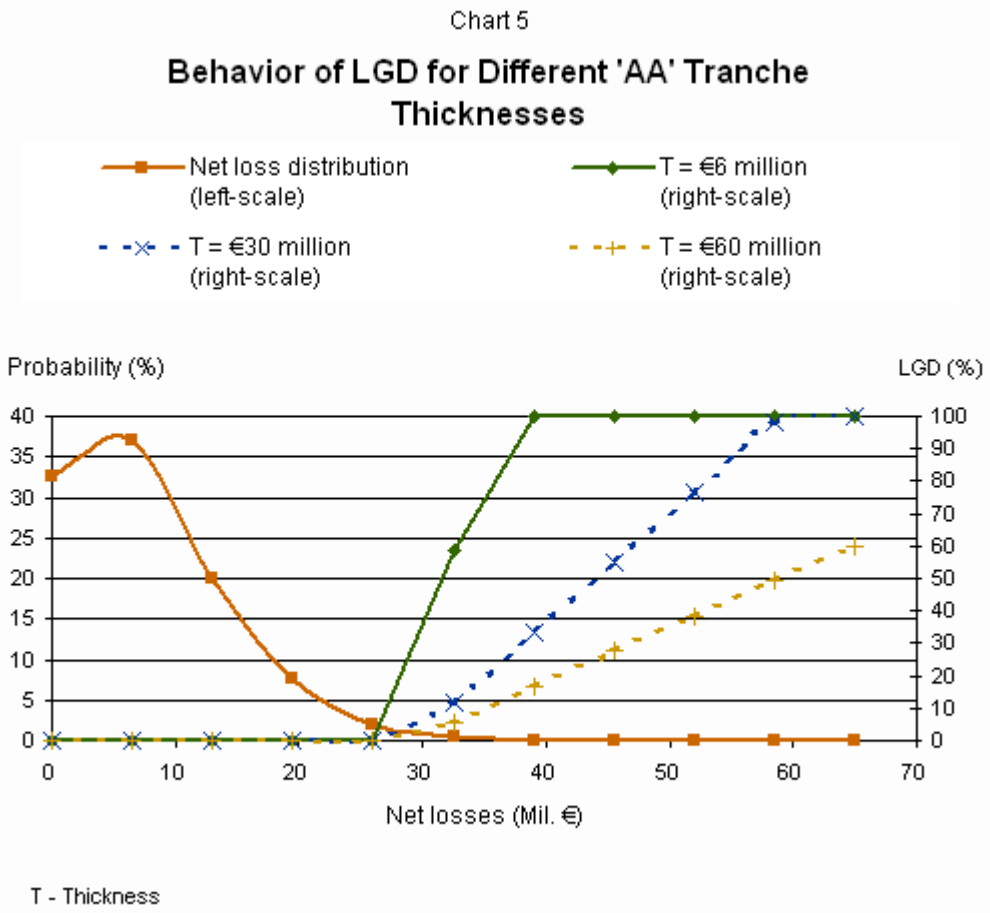
100 'A' corporate names, €10 million notional exposure per name, and 35% recovery



AP - Attachment point
DP - Detachment point

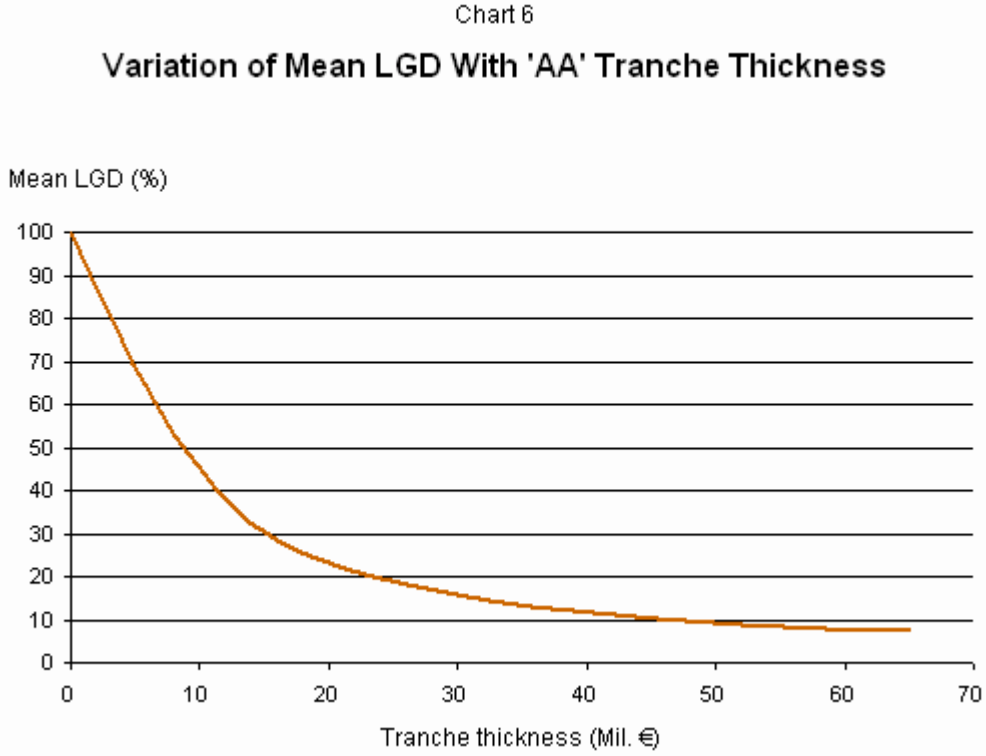
It is important to remember that the examples above show the relationship between tranche thickness and LGD for specific net losses only. In the simulation, a wide range of portfolio net losses is generated with different probabilities, resulting in a wide range of possible tranche losses. Chart 5 illustrates the behavior of the LGD as the thickness of the above 'AA' tranche is varied. Note that in all cases the LGD starts at 0%, and then increases rapidly once the attachment point is breached. It plateaus at 100%, when the detachment point is reached.

Chart 5



It is also possible to calculate the mean LGD across all net losses for different tranche thicknesses by weighting the LGD values by the relevant net loss probabilities. This leads to the curve shown in chart 6, which illustrates the inverse relationship between LGD and tranche thickness.

Chart 6



Determining Attachment Points for the CDO Squared Transaction

Creating the CDO Squared Net Loss Distribution

In each iteration of the model, a net loss is generated for each underlying CDO tranche according to the approach described above. For standalone assets, net losses are generated in the same way as those of the underlying corporate assets. However, as can be seen from chart 1, net losses from standalone assets are passed through directly to the CDO squared transaction.

The net losses from CDO and ABS tranches are simply added together at the CDO squared level to produce the total net loss to the CDO squared transaction in each iteration. Across all iterations, it is therefore straightforward to build up a probability distribution of net losses, ranging from zero to 100% of the total principal balance of the CDO squared transaction.

Setting the CDO Squared Attachment Point

An attachment point can then be determined for the CDO squared transaction in an analogous fashion to a single-tranche synthetic CDO — the probability of exceeding a certain level of net losses is equated to the probability that a corporate bond of a given maturity and rating will default. For example, a five-year transaction would require a 'AAA' attachment point, such that the probability of exceeding the attachment point is less than or equal to 0.284%, Standard & Poor's idealized five-year corporate 'AAA' default probability. As with synthetic CDOs, this attachment point is then multiplied by the relevant adjustment factor.

Case Study

There are two main types of synthetic CDO squared transactions that Standard & Poor's has rated over the second half of 2003 — "pure" CDOs of CDOs and CDOs of CDOs plus standalone ABS assets. The following case study synthesizes the themes outlined in previous sections, using two hypothetical CDO squared transactions.

Case 1: CDO of 25 CDOs

In this synthetic CDO squared transaction, all of the underlying assets are CDOs — there are no standalone assets. It is a five-year transaction ultimately referencing investment-grade corporate names, summarized in table 1.

Table 1 Portfolio Characteristics - Case 1	
Notional of CDO squared transaction (Mil. €)	250
Number of CDO tranches	25
Number of standalone ABS	0
Total number of underlying assets	2,500
Notional of asset portfolio (Bil. €)	25
Number of corporate names	600
Average number of CDO tranches referencing each name	4.17
Number of industries	10
Average industry concentration (%)	13.33
Number of countries	30
Average country concentration (%)	3.33
Weighted-average rating	BBB
Term (years)	5

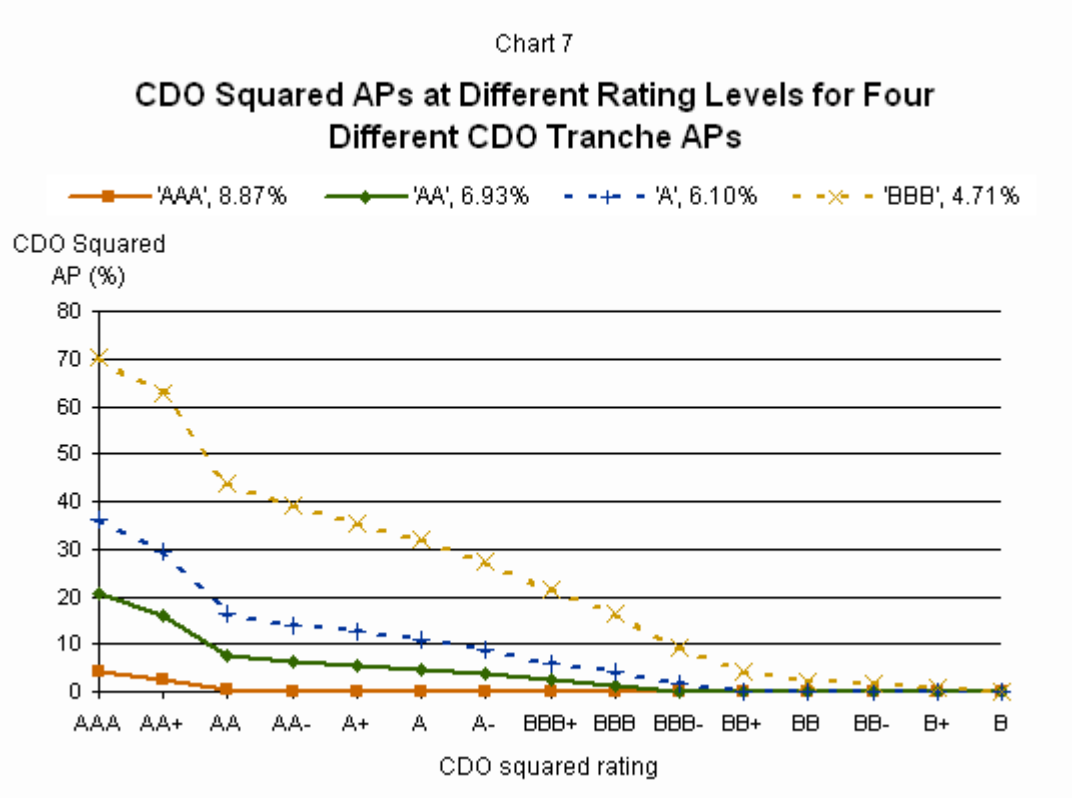
The overlap of corporate names within the underlying CDO tranches is considerable — each corporate name appears, on average, in 4.17 of the underlying CDO tranches. Also, only 10 industries are represented, so the overall correlation between underlying CDO tranches is quite high, which is reflected in the results below.

The first case study examines the final attachment points for the CDO squared transaction in four scenarios, where the attachment point is varied for the 25 underlying CDO tranches being referenced:

- CDO tranche attachment points are set at the 'BBB' level at 4.71%.
- CDO tranche attachment points are set at the 'A' level at 6.10%.
- CDO tranche attachment points are set at the 'AA' level at 6.93%.
- CDO tranche attachment points are set at the 'AAA' level at 8.87%.

The tranche thickness will be assumed to be 2% in all four scenarios. Thus, the detachment point will be 2% above the attachment point. So in the example above, the detachment point at the 'BBB' level would be 6.71%, at the 'A' level 8.10%, and so on.

The final attachment points for the CDO squared transaction are shown in chart 7 .



As expected, the required attachment points for the CDO squared transaction fall as the attachment points for each of the underlying CDOs are increased. Again, higher attachment points for the underlying CDOs mean that fewer losses are passed through to the CDO squared transaction. The 'AAA' attachment point for the CDO squared transaction decreases to 4.05% from 70.27% as the attachment points for the underlying CDO tranches are increased to the 'AAA' level from the 'BBB' level.

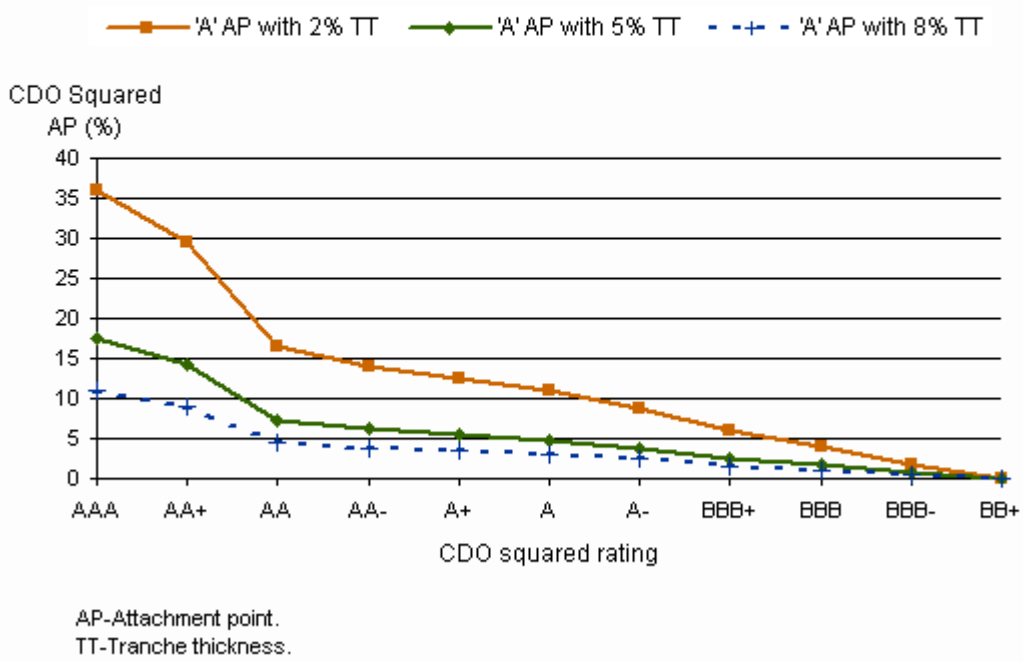
Another way to look at this transaction is to consider the effect of increasing the thickness of the underlying CDO tranches. Three scenarios are examined, all with the CDO tranche attachment point at the 'A' level, but with a:

- Tranche thickness of 2%;
- Tranche thickness of 5%; and
- Tranche thickness of 8%.

The first scenario, involving a thickness of 2%, is the same as for the 'A' attachment point in chart 7 above. In this case, as the tranche thickness is increased — first to 5%, then to 8% — the CDO squared attachment point decreases. For example, the 'AAA' required attachment point in chart 8 for the CDO squared transaction falls to 10.97% from 36.04% as the tranche thickness is increased to 8% from 2%.

Chart 8

Effect on CDO Squared AP as 'A' CDO Tranche Thickness is Increased



AP-Attachment point.
TT-Tranche thickness.

Case 2: CDO of 10 CDOs and 90 Standalone ABS
This transaction contains a mixture of CDO and ABS assets. It is a five-year transaction referencing investment-grade corporate names, as per table 2. The ABS assets are all rated 'AAA'.

Table 2 Portfolio Characteristics - Case 2	
Notional of CDO squared transaction (Mil. €)	5
Number of CDO tranches	10
Number of standalone ABS	90
Total number of underlying assets	1,090
Notional of asset portfolio (Bil. €)	227.75
Number of corporate names	330
Average number of CDO tranches referencing each name	4.22
Number of industries	36
Average industry concentration (%)	2.56
Number of countries	18
Average country concentration (%)	5.56
Weighted-average rating	BBB
Term (years)	5

As before, the overlap in the underlying pools of corporate names is quite high — each corporate name appears, on average, in 4.22 of the underlying CDO tranches. There is more diversity across industries, however, as well as the presence of ABS assets.

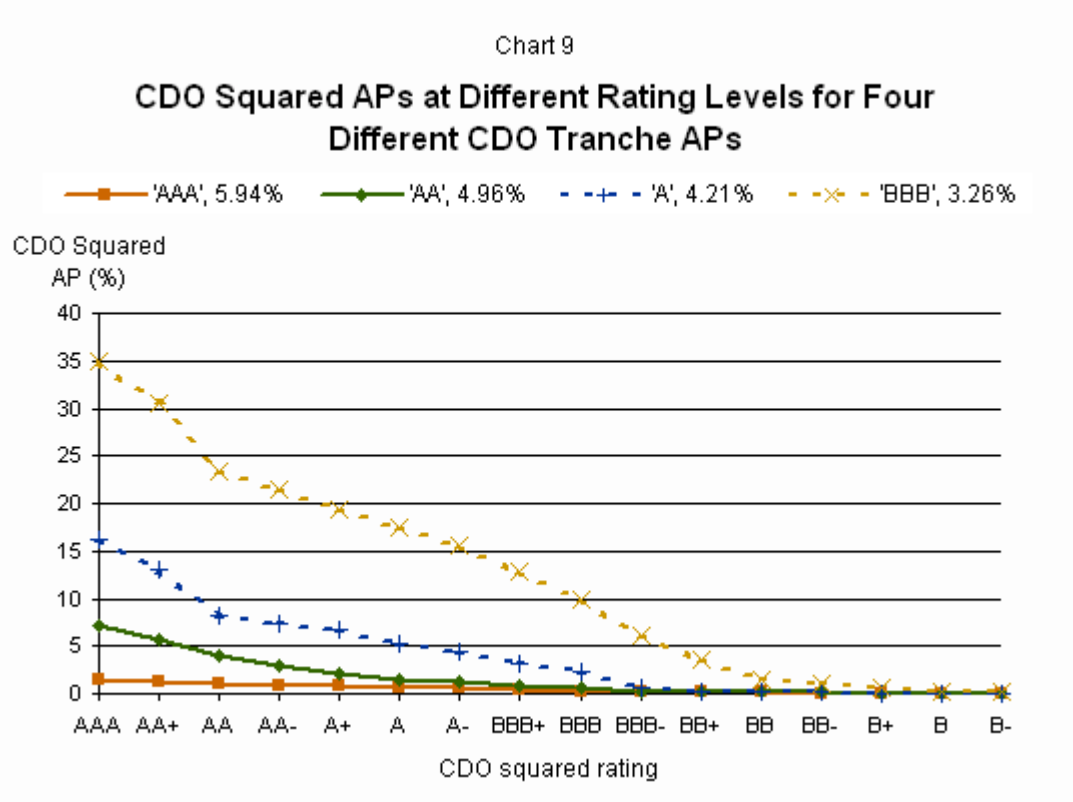
The first set of results shows the attachment points for the CDO squared transaction, using four scenarios for the underlying CDO tranche attachment points. The tranche thickness is assumed to be 1% for each of the 10 CDO

tranches, with CDO tranche attachment points of:

- 3.26% at the 'BBB' level.
- 4.21% at the 'A' level.
- 4.96% at the 'AA' level.
- 5.94% at the 'AAA' level.

The attachment points for the CDO squared transaction, including the impact of both the underlying CDO tranches and the standalone ABS assets, are shown in chart 9.

Chart 9



Again, as the attachment points for each of the underlying CDO tranches are increased, the attachment points for the CDO squared transaction decrease. The 'AAA' attachment point for the CDO squared transaction decreases to 1.48% from 35.02% as the underlying attachment points are increased to the 'AAA' level (5.94%) from the 'BBB' level (3.26%).

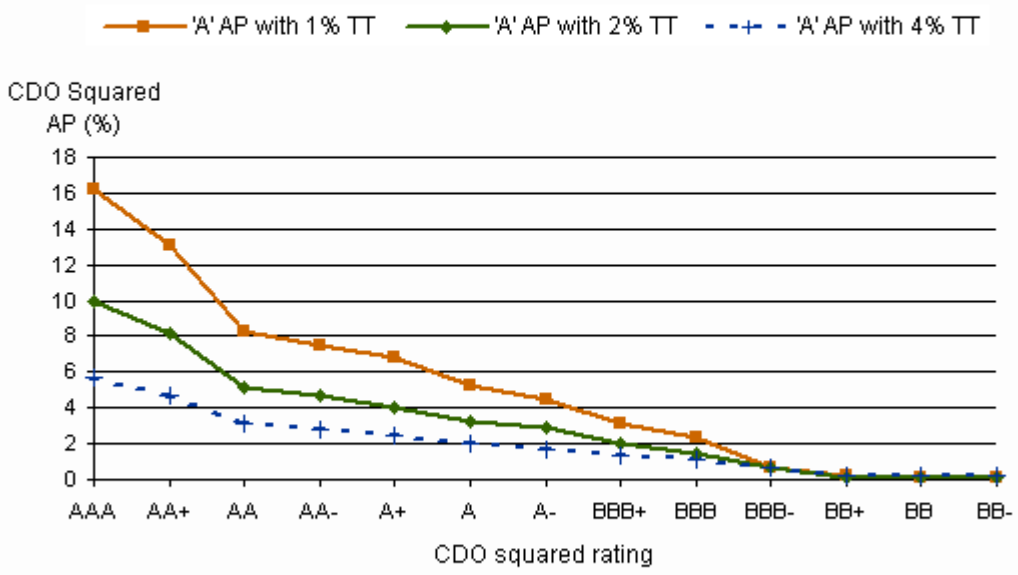
The effect of altering the tranche thickness of the underlying CDOs may also be observed (the ABS remain unaltered). Three scenarios are examined, all with the attachment point at 'A' but with a:

- Tranche thickness of 1%;
- Tranche thickness of 2%; and
- Tranche thickness of 4%.

The first scenario, involving a thickness of 1%, is the same as for the 'A' attachment point in chart 9 above. In this case, as the tranche thickness is increased — first to 2%, then to 4% — the CDO squared attachment point decreases. For example, the 'AAA' required attachment point in chart 10 for the CDO squared transaction falls to 5.69% from 16.20% as the tranche thickness is increased to 4% from 1%.

Chart 10

Effect on CDO Squared AP as 'A' CDO Tranche Thickness is Increased



AP-Attachment point.
TT-Tranche thickness.

This article has shown that the drill-down approach provides an accurate and transparent method of analyzing CDO squared transactions. Standard & Poor's expects the popularity of the approach to increase with the continued structural innovation within the leveraged single-tranche CDO market.

Appendix: CDO Evaluator

Standard & Poor's CDO Evaluator is a "Monte Carlo" simulation tool for the credit risk analysis of synthetic and cash flow CDO transactions. The first version of the Evaluator was released in November 2001. Since then, several new versions have been released, featuring a variety of enhancements to Standard & Poor's CDO risk assessment.

The core of the Evaluator is the simulation of the probability distribution of defaults of a portfolio, which may contain corporate bonds, corporate/SME (small and midsize enterprise) loans, ABS, CDO tranches, or sovereign bonds. For non-emerging market portfolios, the simulation proceeds in six stages:

1. A probability of default is determined for each asset in the portfolio, based on its rating and (except ABS) maturity. This probability is converted into a "z-score" for ease of computation.
2. A correlation matrix is determined for the entire portfolio, based on the industry sectors and regions of each pair of assets. For example, in the case of two corporate names in the same country, Standard & Poor's currently assumes a uniform asset correlation of 0.3 within a sector, and zero between sectors.
3. In each iteration of the CDO Evaluator, a correlated, normally distributed random number is generated for each asset, based on the above correlation matrix. This correlated random number is compared with the z-score of the asset to determine whether the asset defaults in that iteration.

4. Across all iterations, a distribution of potential defaults is created by recording the number and principal amount of defaults that occur in each iteration.
5. A quantile is determined from the above distribution, depending on the desired rating and portfolio maturity. For example, assuming a 'AA' rating and a 10-year maturity, the required quantile of the default distribution will be approximately 2% (i.e., the 10-year Standard & Poor's 'AA' idealized default probability).
6. The above quantile is multiplied by a rating-dependent adjustment factor to give the scenario default rate (SDR). For example, the 'AA'/'AAA' adjustment factors are 1.11 and 1.20, respectively.

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