What is behind OAS measures?

By Alex Levin

While skimming the *Divide and Conquer* trilogy someone asked me: "You don't like the OAS, do you?" The answer is, yes, we do like OAS and generally view it as an application of the option pricing theory. Our idea to replace the OAS measure with prOAS is a proposal not to rely solely on statistical prepayment models and to reflect their potential biases as well as market fears using so-called risk-neutral prepayments instead. Once this is done, the OAS methodology applies. Let us revisit the OAS concept, and provide its comprehensive power.

Defining the OAS rigorously

"OAS is a spread added to random discount rates generated by Monte-Carlo paths to equate averaged present value of contingent cash flows to true market price…" — does such a definition look familiar? It can be found in many textbooks and articles on MBS valuation, but sounds like a mathematical artifact having no economic meaning or financial consequence. What about usual callable bonds that are priced with OAS but without random simulations?

It does not surprise me that practitioners whose only knowledge of OAS is "some spread to plug in" don't gain a deeper understanding and look for alternative measures, often reverting to the antiquated total return analysis. To them I can only exclaim, "OAS is total return!" The following two facts redefine OAS as a total return measure, expected or guaranteed.

<u>Definition/Fact 1.</u> OAS is equal to the **expected** return of a dynamic asset, adjusted for the interest rate risk minus the horizon-matching benchmark (risk-free) rate.

Suppose an MBS is priced off the LIBOR and has an OAS of 50 basis points. We want to assess the total return over a 1-yr horizon. If the 1-year LIBOR is, say, 3%, then our MBS is expected to return 3.5% on a risk-adjusted basis. Its actual return will be random but the mathematical expectation is known.

<u>Definition/Fact 2.</u> OAS is a **guaranteed** annualized earning of a perfectly hedged, perfectly funded dynamic asset.

This statement, unlike the previous one, guarantees the total return level under certain conditions. Suppose we fully fund and fully hedge the MBS in the previous example using LIBOR instruments. Then we will earn 50 bps of par annually, no matter what, for every interest rate scenario. Note that we did not add a word on risk adjustment in Definition/Fact 2. This is because a fully hedged portfolio is interest rate neutral and should bear no risk premium.

Between these two key facts, one can make rigorous return assessments in virtually any combination of investment situations. The only important binding constraint is that the model of randomness employed by an OAS model is adequate.

<u>Question 1.</u> Suppose someone starts a fund by investing in an asset that is fairly priced at 50 bp OAS to LIBOR. The asset is funded by 10% of one's own capital with the rest of their position

(90%) being fully funded via LIBOR borrowings and perfectly hedged. What can be said of the total return on equity (ROE) assuming a 1-year horizon and 3% of the LIBOR rate?

Question 2. Modify Question 1 assuming the fund is 100% hedged, keeping the same (10 + 90) borrowing structure.

Answers to these two questions can be found at the end of this article.

What does prOAS bring to the table?

Only an adjustment for prepay model risk. Hence, one can use prOAS in lieu of OAS in all statements above. "Risk-adjustment" and "perfect hedging" are now understood with respect to both usual interest rate risk and prepay model risk. Should we decide to further adjust for credit risk, we would do similarly, etc.

Option-adjusted effective duration (OAD)

Effective duration is measured by stressing the yield curve immediately by a fairly sizable step (say, 50 basis points) using the parallel pattern. Both assumptions are questionable. How likely can a 50 basis point shock occur on a single day? How often do we observe all points of the curve changing by an identical amount?

Let us not be distracted by the assumptions. The step selection actually matters a little. Duration assessment using the three-point stress test is fairly accurate — the error is due to the third derivative term (not even convexity). The purpose of shocks is not to simulate real-life dynamics, but to measure the price derivative. It is apparent that once the derivative is measured, it can serve for practical purposes well beyond dubious assumptions. One important statement is made below.

Fact 3. As trivial as it sounds, the product of OAD and interest rate volatility is equal to the corresponding price volatility. The same measure can serve as annualized volatility of total return measured over infinitesimal horizon.

If 100 bp/yr is the absolute volatility of interest rate, then a 4-yr duration MBS will have a 4% annualized price volatility. In the example that immediately follows Definition/Fact #1, the expected 1-yr return claims to be 3.5%. If we measure the return over a very short period of time and then annualize it, the total return can be expressed as 3.5% ± 4.0%.

OAD measurement convention that relies on parallel shocks can be easily modified to include durations to interest rate factors. The total volatility is produced by measuring volatility terms due to each factor and then adding them up via Pythagorean math.

Therefore, OAD and its possible modifications point to the total return uncertainty.

Option-adjusted effective convexity (OAC)

Everyone knows that convexity measures changes in duration, but does it have a financial meaning on its own? Yes, it does. OAC is responsible for "diffusion return," i.e. a systematic component of total return that is caused by interest rate volatility. Hence, convexity contributes both to return and, through duration, to risk.

Speaking formally, assets with higher ("better") convexity are expected to have better returns — given everything even. However, this last pre-requisite almost never holds true in real life. For example, a 10-yr zero-coupon bond has a convexity of +1 whereas a 1-yr bill has a convexity of +0.01. Risk-neutral return expectations are identical for these instruments because the dynamics of interest-rates should preclude arbitrage. In usual arbitrage-free models, future rates evolve progressively above the forward rates, thereby compensating for the convexity difference.

Convexity and Vega

Vega measures price elasticity to change in volatility. Every option has a positive Vega; hence, embedded option bonds should have a negative Vega, but not necessarily a negative convexity. Recall that option-free bonds have positive convexity due to the convex nature of discounting. The presence of an option that belongs to the issuer should reduce the convexity of the investor's position, possibly without reaching a negative level. Corporate bonds with remote or deep out-of-the-money call schedules may still have positive convexity; the same applies to high-premium MBS.

It is known that at the money (ATM) options have both the highest convexity (Gamma) and the highest Vega. However, short-dated options usually have small Vega, whereas long-dated options have small Gamma regardless of the strike. These facts are often used to conceptualize and structure a hedge strategy. With option straddles (put + call struck at-the-money) one can control Gamma (short-dated straddle) and Vega (long-dated straddle) separately without interfering with Delta (an ATM straddle has zero Delta).

AD&Co definition of Vega

Since April 1, 2005, we started to compute Vega as price elasticity to the overall volatility scale (calibrated to swaptions or assigned by user). A Vega of -4 means that a 1% increase in the overall volatility scale leads to a 4 basis point drop in price. One apparent advantage of this definition is its versatility — the rate model selection or volatility quotation becomes less relevant. Indeed a 1% increase in the Black-Scholes level (e.g. from 20% to 20.2%) is about equal to a 1% increase in absolute volatility level (e.g. from 90 bps to 90.9 bps). Hence, we should expect to arrive at comparable Vegas for various interest rate models.

For the sake of programming efficiency, we apply volatility shocks to the internally built short-rate volatility function (or constant), but not to the Black-Scholes matrix itself. However, pricing sensitivity to stressing quoted versus internally built volatility scales should be close enough. Hence, our Vega can be tested and verified by OAS model users.

Much like in the case of OAD, Vega can allow to users to assess the uncertainty in price and return assuming some "volatility of volatility."

Assessing VAR through Greeks

By multiplying various reported Greeks by respective factor volatilities and combining them in the Pythagorean way, one can produce the total price volatility. To become a VAR, it needs to be scaled to the desired investment horizon and confidence level. This simple method, however, relies on normality of risk factors and constancy of Greeks. Hence, it applies only to fairly short horizons.

Here comes the morale: the OAS method delivers a much richer outcome than many users think. It points directly to total return, expected or guaranteed, as well as its dispersion (i.e., risk).

Answers to questions

Question 1. ROE = 3.5% expected (adjusted for risk) + 4.5% guaranteed
Question 2. ROE = 8.0% guaranteed